Stepwise training supports strategic second-order theory of mind in turn-taking games

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Abstract

People model other people's mental states in order to understand and predict their behavior. Sometimes they model what others think about them as well: "He thinks that I intend to stop." Such second-order theory of mind is needed to navigate some social situations, for example, to make optimal decisions in turn-taking games. Adults sometimes find this very difficult. Sometimes they make decisions that do not fit their predictions about the other player. However, the main bottleneck for decision makers is to take a second-order perspective required to make a correct opponent model. We report a methodical investigation into supporting factors that help adults do better. We presented subjects with two-player, three-turn games in which optimal decisions required second-order theory of mind (Hedden and Zhang, 2002). We applied three "scaffolds" that, theoretically, should facilitate second-order perspective-taking: 1) stepwise training, from simple one-person games to games requiring second-order theory of mind; 2) prompting subjects to predict the opponent's next decision before making their own decision; and 3) a realistic visual task representation. The performance of subjects in the eight resulting combinations shows that stepwise training, but not the other two scaffolds, improves subjects' second-order opponent models and thereby their own decisions

Keywords: decision making, second-order theory of mind, opponent modeling, scaffolding, turn-taking games, sequential games, centipede, strategic reasoning, perfect-information games

1 Introduction

Why is it that, while we are often told to put ourselves into another person's shoes, we fail to do so when it's most needed? Consider the Camp David negotiations in 1978. Egypt's President Sadat first presented a tough official proposal, but then wrote a much friendlier informal letter to the mediator, US President Carter. This letter contained his fallback positions about the issues on the table. Even though Carter did not tell the exact contents of this letter to Israel's President, Begin still knew about its existence. Sadat failed to wonder about the crucial question: "Does Begin *know* that Carter *knows* my fallback positions?" Begin used his knowledge by pushing towards Sadat's fallback positions in the further negotiations and he made sure that everyone *knew* that the

Israeli parliament *would not* allow Begin to make any concessions on the Palestinian question, which was ultimately left unresolved in the final Camp David Accords (Telhami, 1992; Oakman, 2002). What was Sadat missing?

Many of our daily social interactions also require the ability to infer another person's knowledge, beliefs, desires, and intentions. For example, if we are trying to sell our house, we are reasoning about whether the potential buyer knows that we already bought a new house. The ability to put ourselves in other people's shoes and reason about their beliefs, knowledge, plans and intentions, which may differ from our own, is called theory of mind (henceforth usually ToM, coined by (Premack & Woodruff, 1978)). To be more specific, we use first-order theory of mind to ascribe a simple mental state about world facts to someone, for example: "Ann believes that I wrote this novel under pseudonym, but in fact I did not." This ability to make correct first-order attributions is apparent around the age of 4 (Wellman, Cross, & Watson, 2001; Wimmer & Perner, 1983). Second-order theory of mind adds an extra layer of mental state attribution, as in: "Bob doesn't know that I know that he is the pseudonymous author of this novel."

Second-order ToM is indispensable in a host of social situations. It is needed in communication to understand politeness, humor, irony, *faux pas*, and lying (Filippova & Astington, 2008; Talwar, Gordon, & Lee, 2007; Hsu & Cheung, 2013). Second-order ToM is also needed to make correct decisions in strategic interactions, for example, to win com-

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petitive games and to propose offers that lead to win-win solutions in negotiations, as has been shown using agent simulations (Weerd, Verbrugge, & Verheij, 2013, 2017). So it is a very useful ability, but it is also very hard, and it appears to be developed rather late in childhood.

1.1 Theory of mind is hard: communication, judgment and decision making

Second-order theory of mind starts to appear between the ages of five and seven, when children start to correctly answer second-order false belief questions such as "Where does Mary think that John will look for the chocolate?" about a story in which John peeked in though a window and saw Mary moving John's piece of chocolate from the drawer to the toybox, while Mary wasn't aware that John had seen her in the act (Perner & Wimmer, 1985; Sullivan, Zaitchik, & Tager-Flusberg, 1994; Arslan, Hohenberger, & Verbrugge, 2017). This ability continues to develop during adolescence. Typical adults often display a reasonable understanding of up to fourth-order mental state attributions in stories, e.g. "Alice thinks that Bob doesn't know that she knows that he knows that she sent him an anonymous Valentine card" (Kinderman, Dunbar, & Bentall, 1998).²

However, in social situations, even adults may fail to take other people's perspectives into account. For example in communication, a hearer often doesn't realize that a speaker cannot see some of the objects that the hearer can see (Lin, Keysar, & Epley, 2010). Also, when people are asked to make judgements about other people's problem solving skills, they often fail to apply useful cues such as speed of answering (Mata & Almeida, 2014).

In the "Beauty Contest", all subjects simultaneously have to pick among 1,..., 100 the number they believe will be closest to 2/3 of the average of all subjects' choices. Many people guess 33, which is 2/3 times the average of 1,...,100, thus not taking the perspective of other subjects. Fewer people do use theory of mind and choose at most 22, which is 2/3 times 33 (Nagel, 1995; Camerer, Ho, & Chong, 2015).

Limited use of theory of mind among adults has also been shown in social dilemmas such as the public goods game and the prisoner's dilemma (Colman, 2003; Kieslich & Hilbig, 2014; Rubinstein & Salant, 2016), the trust game (Evans & Krueger, 2011, 2014), and one-shot games such as hideand-seek, matching pennies and the Mod game (Devaine, Hollard, & Daunizeau, 2014a, 2014b; Frey & Goldstone, 2013; Weerd, Diepgrond, & Verbrugge, 2017). In all these games, subjects' decisions in experiments do not fit with

the game-theoretically optimal predicted decision based on common knowledge of rationality, the computation of which requires at least second-order theory of mind.³

1.2 ToM in turn-taking games is hard, too

Particularly difficult tasks requiring theory of mind are *turn-taking games*, also called sequential games or dynamic games. For example, in chess, players have to reason about what their opponents would do in their next move, where the opponent in turn thinks about the first player. Chess is a typical turn-taking game where black and white alternate moves; it is also a perfect- and complete-information game in the sense that both players know the history and rules of the game, in contrast to a game like bridge, in which players cannot see one another's cards. Turn-taking games can be represented by extensive form game trees (see Figures 1 and 2).

So far, a number of turn-taking games of perfect and complete information have been investigated. Games that require second-order theory of mind to make optimal decisions appear to be especially difficult. Whereas 8 year old children already perform at ceiling in second-order false belief story understanding, they start to apply second-order theory of mind in turn-taking games only when they are between 8 and 10 years old, and even then their decisions are on average only slightly better than chance level (Flobbe, Verbrugge, Hendriks, & Krämer, 2008; Raijmakers, Mandell, van Es, & Counihan, 2014; Meijering, Taatgen, van Rijn, & Verbrugge, 2014; Arslan, Verbrugge, Taatgen, & Hollebrandse, 2015).

Even adults are slow to take the perspective of the opponent, let alone to accurately model what their opponent thinks about them, in turn-taking games such as the centipede game and sequential bargaining (Johnson, Camerer, Sen, & Rymon, 2002; Bicchieri, 1989; McKelvey & Palfrey, 1992; Kawagoe & Takizawa, 2012; Ghosh, Heifetz, Verbrugge, & de Weerd, 2017; Bhatt & Camerer, 2005; Camerer et al., 2015; Nagel & Tang, 1998; Ho & Su, 2013). Hedden and Zhang (2002) found that participants on average start with a default, myopic (first-order) theory-of-mind model of the opponent, though the depth of such mental model may be adjusted according to their opponent?s type upon their continued interactions. Zhang, Hedden and Chia (2012) further manipulated perspective taking and showed (p.567) that there is a cost of a factor of 0.65 in terms of likelihood of

¹In these experiments, third and higher orders of ToM hardly offer any additional payoff over and above second-order theory of mind.

²People with autism spectrum disorder undergo a slower development of theory of mind in general and recursive false belief understanding in particular; their perspective-taking difficulties often remain into adulthood (Happé, 1995; Baron-Cohen, Jolliffe, Mortimore, & Robertson, 1997; Kuijper, Hartman, Bogaerds-Hazenberg, & Hendriks, 2017).

 $^{^3}$ In behavioral economics, recursive modeling of other people's possible future decisions is often modeled through iterated best-response models such as level-n models (Nagel, 1995; Kawagoe & Takizawa, 2012; Ho & Su, 2013) and cognitive hierarchies (Camerer et al., 2015). These models are similar to each other and to our orders of theory of mind in that someone's level of sophistication is said to be k+1 if he considers the others to be of level (at most) k. The main differences between the models are in the definitions of the ground level (zero-order) and in whether agents are assumed to have a fixed level or to adapt their level to the opponent's behavior.

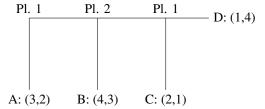


FIGURE 1: Decision tree for an example turn-taking game in which Player 1 chooses first; if he chooses to go right, then Player 2 chooses, and if Player 2 also chooses to go right, finally Player 1 decides again. The pairs at the leaves A, B, C, and D represent the payoffs of Player 1 and Player 2, respectively. This payoff structure corresponds to the Marble Drop game of Figure 5 (c). Figure adapted from Figure 2 of (Rosenthal, 1981) and Figure 1 of (Hedden & Zhang, 2002)

engaging in second-order versus first-order theory-of-mind reasoning.

All these empirical results contradict the prescription of game theory that players, on the basis of common knowledge of their rationality, apply *backward induction* (Osborne & Rubinstein, 1994), which we now explain.

Backward induction

In the typical extensive form game tree of Figure 1, backward induction would run as follows. Player 1 is rational, so at the final decision node, he would decide to go down to C, because his payoff of 2 there is larger than his payoff of 1 in D. Therefore, we can replace the final decision node and its two children by the node (2,1). Taking Player 1's rationality into account, at the second decision point, a rational Player 2 would decide to go down to B, because her payoff of 3 there is larger than her payoff of 1 in the node (2,1). So we can replace the second decision point and its children by the node (4,3). Finally, at the first decision point, the rational Player 1, *believing* that Player 2 *believes* that Player 1 is rational, would decide to move to the right, because his payoff of 4 in the node (4,3) is larger than the payoff of 3 that he would receive if ending the game at A.

In summary, reasoning from the end at the right to the beginning of the game, Player 1 deletes non-optimal actions, one by one. Note that the Backward Induction concept requires both players to reason about what would happen at each possible node, even if that node is never reached in practice (Bicchieri, 1989). But do people really do so?

One paradigm to study how people actually reason in turntaking games is the Matrix Game, which will also be the base game in our experiment.

The Matrix Game

The Matrix Game is a turn-taking game between Player 1 and Player 2, introduced by (Hedden & Zhang, 2002). A typical example can be found in Figure 3. Each of four cells

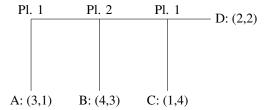


FIGURE 2: Decision tree with a payoff structure corresponding to the Matrix Game of Figure 3

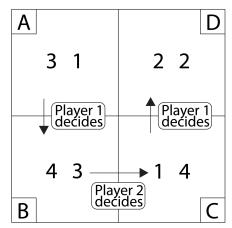


FIGURE 3: The Matrix Game. Figure adapted from Figure 2 of (Hedden & Zhang, 2002)

named A, B, C, and D contains a pair of rewards, so-called payoffs. In each cell, the left number in the payoff pair is Player 1's payoff and the right number is Player 2's payoff if the game were to end up in that cell. For both players, their payoffs in cells A, B, C, D range over the numbers 1, 2, 3, 4 and are all different, that is, there are no relevant payoff ties. Each game starts at cell A. Both players alternately decide whether to stay in the current cell or to move on to the next one. In particular, Player 1 decides whether to stay in cell A or to go to cell B. If the game has not ended yet, Player 2 then decides whether to stay in cell B or to move to cell C. If the game still has not ended, Player 1 finally decides whether to stay in cell C or to move to D. The goal of Player 1 is that the game ends in a cell in which the left payoff is as high as possible, while the goal of Player 2 is that the game ends in a cell in which the right payoff is as high as possible. Thus, both players have a self-interested goal, namely, to maximize their own payoff in the cell in which the game ends up. Unlike many competitive games, it is not a goal to win more points than the opponent and maximize the difference. And, unlike cooperative tasks, it is not a goal to maximize the sum of both players' payoffs.

Still, when making their own decisions, players do have to reason about each other. At cell A in the Matrix Game of Figure 3, Player 1 could correctly reason about a rational

opponent: "Player 2 *believes* that I *intend* to move to cell D at my last decision point, because my payoff there is higher than in cell C. At D, however, Player 2 only wins a payoff of 2, so he *intends* to stay at B in order to get the higher payoff 3. But that is excellent for me, because ending up at B will give me a payoff of 4. Therefore, I decide to move on to B."

Adults have trouble making the correct mental model of the other player (Hedden & Zhang, 2002; Zhang, Hedden, & Chia, 2012): The proportion of games in which subjects made correct second-order predictions about their opponent was only 60%–70%, even at the end of the cited experiments.⁴ Adolescents and young adults have an even harder time playing the Matrix Game if they are sequentially pitted against against both "myopic" (zero-order ToM) and 'predictive' (first-order ToM) opponents (Li, Liu, & Zhu, 2011).

1.3 Supporting strategic second-order ToM

We present an experiment to investigate how adults can best be supported in applying second-order theory of mind in games with three decisions such as the Matrix Game. Advantages of these games are that it is possible to construct games of different levels of complexity in terms of required theory of mind and to develop an idea of subjects' reasoning strategies on the basis of their mistakes.

In order to correctly gauge subjects' level of theory of mind from their decisions, we need to test them in a large number of different game items, so that they cannot find an optimal solution just by pattern recognition alone. Fortunately, for the Matrix Game, it is possible to devise many different payoff distributions that require such second-order perspective taking on the part of Player 1, who wants to make an optimal decision at the start of the game.

Controlling the opponent's decisions

If we want to support people in their second-order theory of mind, it is important to control the strategy and the level of theory of mind used by the opponent: A subject displays second-order theory of mind by making a correct mental model of a first-order opponent. To achieve this, many researchers use a human confederate of the experimenter, who strictly follows pre-determined strategies (Hedden & Zhang, 2002; Zhang et al., 2012; Li et al., 2011; Goodie, Doshi, & Young, 2012). In many studies, subjects play against a computer opponent. In such cases, they are sometimes deceived by a story that they are playing against another human being (Hedden & Zhang, 2002); other times, they are told that they are playing "against four different players" (Devaine et al., 2014a); at the most honest end of the spectrum, subjects are told that they are playing against a very smart computer opponent, possibly including information about the opponent's rationality or level of perspective taking (Hedden & Zhang, 2002; Weerd, Verbrugge, & Verheij, 2017; Ghosh et al., 2017). We choose the honest procedure here.⁵

Scaffolding new skills

Wood and colleagues introduced the term "scaffolding" to describe the types of support that an adult or expert could give to a child that initially is not able to solve a problem or perform a task: "This scaffolding consists essentially of the adult 'controlling' those elements of the task that are initially beyond the learner's capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence" (Wood, Bruner, & Ross, 1976, p. 90). Recently, scaffolding has also been used in the context of adults learning new skills (Clark, 1997), as we will do here.6

In our experiment, we attempt to support subjects' strategic second-order theory of mind by three different scaffolds:

- Stepwise training (compared to Undifferentiated training);
- 2. Prompting them for predictions of their opponent's next decision (compared to No prompts);
- 3. using a more intuitive visual Task representation called Marble Drop (compared to the Matrix Game).

In the next section, we explain the three scaffolds in detail as well as the *a priori* reasons why they may be helpful. We also discuss the methods of the experiment. In Section 3, we present the results and explain which factors (alone or together) influence the performance of decision makers in the two-player turn-taking games. In Section 4, we zoom in both on the kinds of mistakes subjects made and on the reasoning strategies underlying their optimal decisions. We conclude with some suggestions for possible future experiments on de-biasing decision makers.

2 Method

2.1 Subjects

Ninety-three first-year psychology students (63 female) with a mean age of 21 years (ranging between 18 and 31 years) participated in exchange for course credit. All subjects had normal or corrected-to-normal visual acuity. One subject was excluded due to an error in the experimental setup.

⁴By chance alone, the proportion would have been 50%, given that games had been balanced in terms of 'stay' or 'go' predictions.

⁵Hedden and Zhang (2002) have compared different treatments and have shown that inclusion of a cover story about the opponent being a human or a computer did not affect subjects' performance. Interestingly, Goodie and colleagues (2012) state that people may actually be able to take a second-order perspective but that they do not use it, simply because they have such low opinions of their fellow human beings' level of reasoning.

⁶Thus, scaffolding is relevant for those skills that are in a child's "zone of proximal development", as defined by (Vygotsky, 1987). For a Vygotskian theory on scaffolding in pre-school children's development of first-order theory of mind, see (Fernyhough, 2008).

2.2 Design

The experimental design comprised three factors: training, prompting predictions, and task representation. All factors were administered to 93 subjects in a $2 \times 2 \times 2$ between-subject design, with Stepwise/Undifferentiated training crossed with Prompt/No-Prompt crossed with Marble-Drop/Matrix-Game. The experiment consisted of three blocks: one training block followed by two test blocks. We now proceed to explain the three factors training, prompting predictions, and task representation in detail, and to explain a priori why each of the three manipulations should provide the kind scaffolding discussed in the Introduction, to support decision makers in their second-order theory of mind and in making optimal decisions in the games.

2.3 Scaffold 1: Training

The training block was included to familiarize subjects with the rules of the games. Subjects were randomly assigned to one of two training procedures. In one training procedure, subjects were presented with Hedden and Zhang's (2002) 24 original training games (see Figure 4; top panel). These so-called "trivial" training games are easier to play than truly second-order games such as in the game on the bottom right in Figure 4, because Player 2 does not have to reason about Player 1's last possible decision: Player 2's payoff in B is either lower or higher than both his payoffs in C and D. For example, in the game in the upper panel of Figure 4, to make an optimal decision, it suffices for the subject to make the following correct first-order attribution: "The other player intends to move from B to C, because his goal is to earn as many points as possible, and in both C (3 points) and D (4 points), he will receive more points than if he stays in B (2 points)."

This training procedure will henceforth be referred to as *Undifferentiated training*, because all 24 training games of this type are of the same kind: they have three decision points while only requiring first-order theory of mind. Possibly, these trivial games in the training block made the change to the test blocks difficult for the subjects of (Hedden & Zhang, 2002), who were suddenly required to perform second-order perspective taking in order to make optimal decisions in the Matrix Games.

In the other training procedure, which we name *Stepwise training*, subjects were therefore presented with three blocks of games that are simple at first and become increasingly complex with each block, subsequently requiring zero-order, first-order, and second-order theory of mind to find the optimal decision (Figure 4; bottom panel). More precisely, the first training block of Stepwise training consisted of 4

games with just one decision point. These games are called zero-order games, because they do not require application of ToM. The second training block consisted of 8 games with two decision points. These games require application of first-order ToM, for example, in Figure 4 (middle of bottom panel): "The opponent *intends* to move from B to C, because his *goal* is to earn as many points as possible and 4 > 2".

The third training block consisted of 8 games with three decision points that require application of second-order ToM, because the subject has to reason about the other player, and take into account that the other player is reasoning about them. The subject could make the following second-order attribution to the opponent: "The opponent *thinks* that I *intend* to move from C to D, because he *knows* that my *goal* is to earn as many points as possible, and 2 > 1". The pay-off structures in the 8 games of this third training block were chosen in such a way that they were diagnostic of second-order ToM reasoning, in the sense that first-order ToM reasoning does not lead to an optimal solution for them, unlike in the Undifferentiated training games. (See Appendix C for more explanation of our selection of payoff structures.)

We hypothesize a priori that the Stepwise training procedure provides scaffolding to support the representation of increasingly more complex mental states. Stepwise introduction, explanation, and practice of reasoning about each additional decision point helps subjects integrate mental states of increasing complexity into their decision-making process. Support for this hypothesis is provided by studies of children, who learn the orders of theory of mind one by one. Five-year old children have already had multiple experiences in daily life with first-order perspective taking. For them, secondorder false belief questions are in the zone of proximal development. They require exposure to a number of such tasks, in which they are asked to subsequently answer zero-order, first-order and second-order questions about a story and to justify their answers. In this way, they can be trained in second-order perspective taking (Arslan et al., 2015; Arslan, Taatgen, & Verbrugge, 2017).

For adults, second-order ToM in turn-taking games is similarly in their zone of proximal development: they already solve zero-order and first-order versions very well. We surmise that Stepwise training helps them to completely master zero- and first-order ToM in these games before building on these answers in their second-order perspective-taking. Like the developmental studies, our training phase also includes asking subjects to explain what they should have done and why in case they made a non-optimal decision.

2.4 Scaffold 2: Prompting predictions

The second factor, prompting subjects for predictions, was manipulated in the first Test Block. Hedden and Zhang (2002) prompted their subjects to predict Player 2's decision (in cell B, see Figure 1), before making their own decision.

⁷See Appendix B for the numbers of subjects in each of the 8 resulting groups and for the aggregate numbers of subjects resulting from the three binary factor divisions.

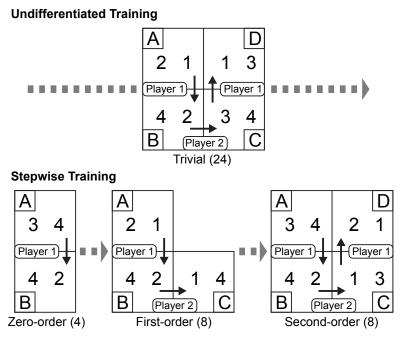


FIGURE 4: Schematic overview of the Undifferentiated and Stepwise training procedures for the Matrix Game. Undifferentiated training consists of 24 different so-called trivial games (top panel, see Subsection 2.3 for explanation). Stepwise training consists of 4 zero-order games, 8 first-order games, and 8 second-order games. The actual 20 training items all had different payoff distributions (bottom panel).

Thus, subjects were explicitly asked to take the other player's perspective, and we hypothesize *a priori* that these prompts help subjects to actually use second-order theory of mind attributions such as "the opponent *thinks* that I *intend* to stay at C" in their decision-making process.

We tested this hypothesis in the two Test Blocks of 32 second-order games each. In the first Test Block, we asked half of the subjects, randomly assigned to the Prompt group, to predict Player 2's move before making their own decision. Subjects assigned to the No-Prompt group, in contrast, were not explicitly asked to predict Player 2's move; they were required only to make their own decision throughout the experiment. The second Test Block was added to test whether prompting had long-lasting effects on performance. No subject was asked to make predictions in the second test block, and performance differences between the Prompt group and the No-Prompt group would indicate lasting effects of prompting.

2.5 Scaffold 3: Visual Task representation

The third and final factor that we manipulated is the visual task representation. Before the training phase started, subjects were randomly assigned to one of two task representations, which did not change anymore during the remainder of the experiment. One of the representations was the Matrix Game (Hedden & Zhang, 2002), which is sometimes

criticized for being very abstract and therefore difficult to understand for subjects (Goodie et al., 2012). We therefore devised a second representation, henceforth referred to as Marble Drop, which is similar to the extensive-form game trees so as to clarify the recursive structure of the decision-making problem by displaying more intuitively who decides where and what the consequences of each decision are (Figure 5) (Meijering, van Rijn, Taatgen, & Verbrugge, 2012).

Figure 5 depicts examples of zero-order, first-order, and second-order Marble Drop games. A white marble is about to drop, and its path can be manipulated by opening the left or right trapdoor at each decision point. Player 1's goal is to let the white marble drop into the bin containing the darkest possible marble of his/her target color (blue in these example games), by controlling only the blue trapdoors. Player 2's goal is to obtain the darkest possible orange marble, but Player 2 can only control the orange trapdoors. The marbles are ranked from light to dark, with darker marbles preferred over lighter marbles, yielding payoff structures isomorphic to those in matrix games. Each time, opening the left trapdoor provides access to a single bin, thus ending the game ("to stay"). Opening the right trapdoor ("to go") allows the marble to move to the right to a new decision point, a pair of trapdoors of the other color, where the other player decides. The subject is always Player 1, deciding which of the first pair of trapdoors to open.

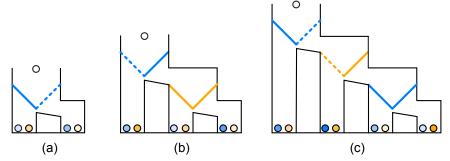


FIGURE 5: Examples of zero-order (a), first-order (b), and second-order (c) Marble Drop games between Player 1 (blue) and Player 2 (orange). The dashed lines in the figure represent the optimal decisions. (See Subsection 2.5 for explanation.)

For example, in the game of Figure 5, panel (c), Player 1 could reason as follows: "Suppose I were to open the right blue trapdoor at my first decision point. Then after that, Player 2 would not open his right orange trapdoor, because he *knows* that if he did, I would then *plan* to open the left blue final trapdoor, which would give him the lowest possible payoff, the lightest orange marble. So he would open the left orange trapdoor, which would in turn give me my highest possible payoff, the darkest blue marble. So, let me open the right blue trapdoor at the top."

We designed the game in such a way that experience with world physics, in particular with marble runs in childhood, would allow subjects to easily imagine how the marble would run through a game. Moreover, the interface of the game was designed to support subjects to quickly see who could change the path of the marble at which point in the game, because the trapdoors were color-coded according to who got to decide where and the target color of that player. Finally, we implemented the experiments with Marble Drop in such a way that the subjects could see the marble drop down and, after their initial decision at the first pair of trapdoors, they could visually follow the marble as it coursed through the Marble Drop device on the screen.

We hypothesize *a priori* that the new visual task representation of Marble Drop provides scaffolding that supports correct use of second-order theory of mind and thereby leads to better decisions. A similar approach has been shown to support subjects in learning other dynamic games, such as Number Scrabble, which is formally equivalent to the well-known game of Tic-Tac-Toe, whilst subjects perform significantly better in the latter game (Michon, 1967; Simon, 1979; Weitzenfeld, 1984).8

Importantly, Marble Drop is game-theoretically isomorphic to Matrix Games and thus requires essentially the same reasoning (see next subsection). Instead of using numerical payoffs, as commonly used in experimental games, we chose colored marbles to counter numerical but non-optimal

reasoning strategies towards goals such as minimizing the opponent's outcomes, maximizing the sum of both players' outcomes, or maximizing the difference between Player 1 and Player 2 outcomes.

Equivalence of Matrix Game, Marble Drop and extensive form game trees

All three game representations, namely the Matrix Game, Marble Drop and the classical extensive form game trees, turn out to be game-theoretically equivalent: the backward induction strategy yields the same intermediate and final results for them. The three representations all have terminal nodes, subsequently named A, B, C, and D. Moreover, the temporal order of play is the same: First, Player 1 decides whether to "stay" and end the game at A or to "move" further. Then at the second decision point, Player 2 decides whether to stay and end the game at B or to move further. Finally, at the third decision point, Player 1 decides whether to stay and end the game in C or to move and end the game in D.

Finally, the payoff pairs correspond one-to-one between the representations. For example, the extensive form game tree of Figure 2 has the same payoff structure as the Matrix Game of Figure 3. The payoff structure of the extensive form game tree of Figure 1 corresponds to the Marble Drop of Figure 5 (c), where payoff 1 in the game tree corresponds to the lightest shade of the corresponding player's target color, and payoff 4 to the darkest shade.

2.6 Stimuli

Payoffs

The payoffs in Matrix Games are numerical, ranging from 1 to 4, whereas the payoffs in Marble Drop games are colorgraded marbles that have a one-to-one mapping to the numerical values in the Matrix Games. The colors of the marbles are four shades of orange and blue, taken from the HSV (i.e., hue, saturation and value) space. A sequential color palette is computed by varying saturation, for a given hue and value. This results in four shades (with saturation from .2 to 1) for

⁸Visual presentation has been shown to support other judgment and decision tasks, see (Dambacher, Haffke, Groß, & Hübner, 2016).

each of the colors orange (hue = .1, value = 1) and blue (hue = .6, value = 1).

Payoff structures

The payoff structures of the two-person three-stage turn taking games were adapted from the original game designs of (Hedden & Zhang, 2002). The payoff structures are selected so that the order of ToM reasoning mastered by the subjects can be derived from the set of their first decisions in the experimental games. The total set of payoff structures, balanced for the number of decisions to continue or stop a game, is limited to 16 items. These items are listed in Appendix C, including a detailed discussion of the rationale behind the exclusion criteria.⁹

2.7 Procedure

The 43 subjects in the Marble Drop condition were first tested on color-blindness. They had to be able to distinguish the two colors blue and orange, and to correctly order the four grades of both orange and blue in terms of darkness.

To familiarize them with the rules of the sequential games, all subjects were first presented with a training block that either consisted of Undifferentiated training or Stepwise training. The instructions, which appeared on screen, explained how to play the games and what the goal of each player was. For example, the subjects in the Stepwise training and Marble Drop condition with orange as target color received the following instruction about the first-order training games: "In the next games, there are two sets of trapdoors. You control the orange trapdoors and the computer controls the blue trapdoors. The computer is trying to let the white marble end up in the bin with the darkest blue marble it can get. You still have to attain the darkest orange marble you can get. Note, the computer does not play with or against you." For an example full verbatim instruction set with illustrations, see Appendix A.

The instructions also mentioned that subjects were playing against a computer-simulated player. Hedden and Zhang (2002) have shown that inclusion of a cover story about a purported human opponent did not affect ToM performance. Each training game was played until either the subject or the computer-simulated player decided to stop, or until the last possible decision was made. After each training game, subjects were presented with accuracy feedback indicating whether the highest attainable payoff was obtained. In case of an incorrect decision, an arrow pointed at the cell of the Matrix Game (respectively the bin of Marble Drop) that contained the highest attainable payoff and the subject was asked to explain why that cell (or bin) was optimal for them.

Because the feedback never referred to the other player's mental states, subjects had to infer these themselves.

The training block was followed by two test blocks, which consisted of 32 second-order games each. As mentioned in Subsection 2.4, the procedure for subjects in the Prompt and No-Prompt groups differed in the first test block. Subjects in the Prompt group were first asked to enter a prediction of Player 2's decision before they were asked to make a decision at their own first decision point — between stay-orgo in the Matrix Game, respectively between opening the left or right trapdoor in Marble Drop. Subjects in the No-Prompt group, in contrast, were not asked to make predictions. They were asked only to make decisions. Accuracy feedback was presented both after entering a prediction and after entering a decision, but the arrow pointing to the highest attainable payoff was not shown anymore in the test blocks. The first test block consisted of 32 trials; each of the 16 payoff structures was presented twice, but not consecutively. The items were presented in a different random order for each subject. The second test block, also consisting of 32 games, followed the same procedure for all subjects: They were asked to make their own first decisions only.

3 Results

3.1 Reaction times

Subjects were not instructed regarding the speed of responding: they were asked only to do the task as well as possible. In the two test blocks consisting of 32 items each, subjects were confronted with four occurrences each (in a random order) of the 16 essentially different games (Table 2, Appendix C). Fortunately, the reaction time data strongly speak against the possibility of the subjects having used simple cognitive strategies such as pattern recognition, because the mean reaction time was still more than 8 seconds (M = 8.5, SE = .61) even in Test Block $2.^{10}$

The setup of the prompt condition in Test Block 1 did not allow us to make a more detailed analysis of the decision times, because the reaction times of the decisions that subjects in the Prompt group made after having first made an explicit prediction about the opponent's next decision, were expected to be much lower than the bare decisions of subjects in the No-Prompt group, due to the experimental design. Moreover, for subjects in the Prompt group, we could not gauge from their decision times whether their decisions took shorter in Test Block 2 than in Test Block 1. There-

⁹Note that none of the 16 game items in Table 2 corresponds to a classical centipede (Rosenthal, 1981; Osborne & Rubinstein, 1994) with Nash equilibrium at A and an incentive for Player 1 to signal to Player 2 to cooperate by moving on towards a Pareto-optimal outcome at D.

¹⁰In contrast, (Mehlhorn, Taatgen, Lebiere, & Krems, 2011) did find signs of pattern recognition in their diagnostic reasoning task, indicated by reaction times shorter than 1.5 seconds. For interesting experiments validating that short response times correspond to instinctive reasoning and long response times to slow reflective reasoning in judgment and decision making, see (Rubinstein, 2013).

fore, only the accuracy scores are discussed in the rest of the current study.

3.2 Scaffolding effects

Figure 6 shows the proportions of accurate predictions for each of the conditions in the experiment. The data suggest effects of prompting, training and block, but not necessarily of task representation. To test the effects of the scaffolding manipulations, we analyzed the accuracy data of Test Block 1 and Test Block 2 using binomial mixed-effect models, with subject and problem-ID as random effects (Baayen, Davidson, & Bates, 2008). The training block was excluded from analysis, because the trials differed between the Stepwise and Undifferentiated types of training. Because we were interested in the main effects of our manipulations and the interactions on block, we first fitted a model with Task representation (Marble Drop or Matrix Game), Prompt (i.e., prompting or not for predictions about the opponent's choice at the second decision point), Training (i.e., the type of training: Stepwise or Undifferentiated), and Block (i.e., Test Block 1 or 2) as main effect, and Task representation × Block, Prompt × Block, Training × Block as interaction effects.

Analysis of variance indicates main effects of Training (stepwise better), Prompt, and Block (all with p < .002). The effect of Task (matrix/marble) was not significant. Figure 6 suggests that interactions are also present, but most of these could result from reduction in room for effects to manifest themselves as accuracy approached the ceiling (and in fact reached it for some subjects in some conditions).

Of more substantive interest, Prompt had a larger effect in Block 1 than in Block 2 (p < .001 for the interaction overall, and also for the interaction within each Task); the interaction shows up in Figure 6 as greater slopes (between blocks) for No-prompt than for Prompt. It is a clear cross-over interaction for the Marble game and this cannot be explained in terms of compression of the scale. This effect could be interpreted as a non-lasting effect of Prompt: Prompting subjects helps them in second-order ToM reasoning by breaking up the reasoning steps, but after prompting stops, in Test Block 2, the advantage largely disappears.

3.3 Alternative strategies

One could argue that strictly competitive games — in the sense that a win for one player is a loss for the other and vice versa — are easier and more intuitive for people, in contrast to the current game in which both the subject and the opponent had the self-interested goal of maximizing their own payoff. Even though the subjects were carefully instructed about the objective of the game (see verbatim instructions in Appendix A), it might be that some subjects played competitively in the sense of maximizing the difference, namely

their own score minus the opponent's score. Other subjects might have played cooperatively in the sense of maximizing social welfare, the sum of their and the opponent's scores. Therefore, we checked whether the behavior of a subject was consistent with one of the strategies on the given items, using a moving average of 11 trials. If the strategies could not be discriminated based on the responses, the subject would be assigned to more than one strategy.

As depicted in Figure 7, subjects likely did follow the instructions to be self-interested by maximizing their own payoff and to assume that the opponent was self-interested too; especially in the second half of the experiment, more than 80% could be so classified.

4 General Discussion

The main aim of our experiment was to find out how we can best support adults in the perspective taking needed to make optimal decisions in a three-step two-player game. We chose three manipulations for which we had reason to believe that they would help scaffold subjects' second-order theory of mind, namely 1) Stepwise training, 2) Prompting subjects to make an explicit prediction of the opponent's next choice (that included the opponent's taking the subject's final choice into account) and 3) a less abstract, easier to understand visual task representation, namely Marble Drop.

Subjects clearly benefit from a carefully constructed Stepwise training regime, in which they are first asked to make decisions in a few one-choice versions that introduce the goal and the game representation but do not require any reasoning about an opponent, then a few two-choice versions in which subjects have to use first-order theory of mind to predict the opponent's next (simple) choice between two end-points, and finally a few three-choice versions for which secondorder theory of mind is required. Their accuracy is much higher than for the other half of the subjects, who had gone through a training regime in which all training games were three-choice games with payoffs distributed in such a way that the third choice made no difference for the optimal first and second decisions. For such so-called trivial games in this Undifferentiated training regime, first-order theory of mind was always sufficient.

Prompting subjects to think about the opponent's perspective at the next decision point helps them in making their own optimal decision at the first decision point of the current game. In particular, when explicitly prompted, subjects tend to give correct predictions. However, prompting improves performance mainly in the session in which it is applied, and does not have much lasting effect on the subsequent session without prompts.

Finally, it appears that the Task representation does not really matter: Subjects achieve about the same accuracy scores in the Marble Drop representation as in the more

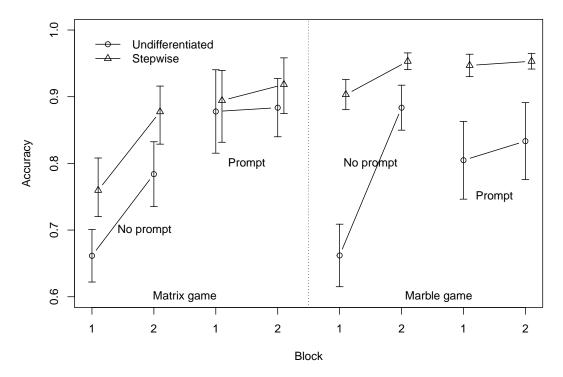


FIGURE 6: Accuracy results for all 8 conditions of the experiment for Test Block 1 and Test Block 2. Error bars represent one standard error.

abstract Matrix Game. This surprised us for several reasons. Firstly, we had carefully designed Marble Drop to be easy to understand and to fit with people's experience as children with games in which marbles drop and slide down devices, partly under control of the child. Secondly, the decision-making literature abounds with examples in which a proper visual representation improves people's accuracy. Finally, several subjects told us in the debriefing how insightful they had found Marble Drop. Apparently, the less abstract visual representation of Marble Drop is not sufficient to support subjects to take the second-order perspective required to make optimal decisions.

4.1 Errors of rationality and theory of mind

Zhang and colleagues argue that, for people to make an optimal first decision in three-move two-player Matrix Games, they need two different capacities: a) perspective taking by recursive theory of mind, to make a correct mental model of their opponent and predict his next decision and b) instrumental rationality, to make an optimal first decision of their own, based both on their mental model of the opponent and an analysis of relevant payoffs. Thus, instrumental rationality errors are made when subjects are not engaged in "fully enacting the possible consequences of their opponent's action and making contingency plans based on these predicted consequences" (Hedden & Zhang, 2002; Jones & Zhang,

2003; Zhang et al., 2012).

4.1.1 Rationality errors

In the Prompt group in Test Block 1, a number of subjects fail to conclude from their correct prediction what their own optimal decision should be. Sometimes subjects make the opposite error: They first make an incorrect prediction about the opponent's next step, but do not to take it into account when making their own, optimal, decision at the first decision point. We follow Hedden and Zhang (2002) in calling both types of mismatch *rationality errors*. Subjects in the Prompt group made rationality errors in 12.6% of trials in the relevant Test Block 1.

If we look more closely at the total number of 191 (out of 1504) trials in our experiment in which a subject made a rationality error — trials with a correct decision but a wrong prediction and vice versa — we see that subjects in those trials tend to end the game by staying in cell A in the Matrix Game or let the marble drop into the leftmost bin of Marble Drop so as to receive the first possible pay-off (n = 122) instead of giving the opponent a chance to play (n = 69). This asymmetry is striking, considering that the numbers of times a subject should stay or go according to the optimal solution prescribed by game theory, had been

¹¹Note that in our experiment, we can check for rationality errors only in the Prompt group in the first test block.

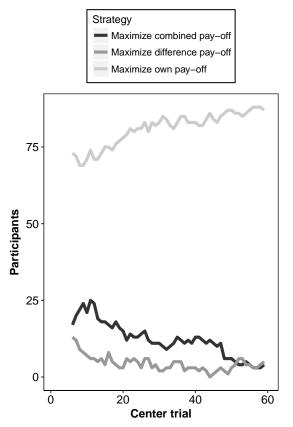


Figure 7: The y-axis shows the number of subjects that are assigned to a specific strategy. On the x-axis is depicted the center trial of the moving average. The "Maximize own payoff" corresponds to the task instructions.

balanced throughout the experiment (Table 2, Appendix C). This finding indicates that a majority, around 2/3 of subjects, use a risk-averse strategy when making a rationality error.

In contrast, in the studies of Zhang and colleagues this "stay in cell A" type of rationality error occurred equally often as the "move to cell B" type. Because all the trials in our test blocks are diagnostic for second-order ToM, the payoff for the player was 3 for the first cell/bin in all such trials in which they "played safe". These subjects would forego an attainable maximal payoff of 4 at a later stage of the game (see lines 9–16 of Table 2, Appendix C). However, this type of rationality error is rationalizable: The subjects could be sure of gaining 3 points when staying at A, while they ran

the risk of gaining only 1 or 2 points if the opponent were to unexpectedly decide differently from their predicted rational decision. In these cases, subjects were not absolutely certain that the opponent would act according to their prediction and preferred to keep matters in their own hands.

There are also important similarities between our experiment and Zhang and colleagues' in terms of rationality errors. Zhang et al. (2012) make a comparison of the ratios of rationality errors in cases in which subjects need only first-order theory of mind in cell B when making their own decision in the role of Player 2, versus cases in which subjects in the role of Player 1 need to make a prediction about their opponent Player 2's decision in cell B, requiring secondorder theory of mind. Interestingly, the ratios of rationality errors that subjects make are almost the same, whether they need to use first-order or second-order theory of mind, and rather low in both cases: decreasing from around 1/5 in the beginning of the 64 test games down to around 1/10 at the end (Zhang et al., 2012). Similarly, in our experiment, rationality errors occurred only around 1/8 of items in Test Block 1.

Therefore, both Zhang, Hedden and Chia and our current study corroborate the earlier results of Hedden and Zhang: The ratios of rationality errors do not significantly differ between the condition in which subjects play standard Matrix Games against a myopic (zero-order ToM) opponent or against a predictive (first-order ToM) opponent (Hedden & Zhang, 2002). Thus, all these studies support the conclusion that instrumental rationality (making your own decision on the basis of your prediction of the opponent's next decision) is distinct from and apparently easier for subjects than the perspective taking required to make a correct mental model of the opponent, requiring second-order theory of mind (Jones & Zhang, 2003).

4.1.2 Errors in recursive perspective taking

In the current experiment, the non-optimal decisions that subjects make are mostly due to difficulties in making a correct model of the opponent. Because we usually did not ask subjects for explicit predictions (only the Prompt group in the Test Block 1), we have to gauge their predictions from their decisions and we cannot provide an analysis of perspective-taking errors in their predictions. However, the patterns of decisions per subject indicate that it was clear to them that the opponent's goal, like the subject's own, was to self-interestedly maximize their own payoff; they did not think, for example, that the goal was to maximize the joint payoff or to maximize the difference between the players' payoffs (see Subsection 3.3).

Our previous cognitive modeling work on Marble Drop indicates that adults, different from children, do take the opponent's goals into account. However, when subjects make non-optimal decisions, their decision patterns over all payoff

¹²The difference between the tendencies of the rationality errors in these two studies may be due to the fact that Hedden and Zhang also used a number of payoff structures in which the subject's payoff in cell A was 2. We did not use these, because we needed to double-balance optimal "stay" and "go" decisions at both the first and second decision points, in order to distinguish whether a subject had a "myopic" or "predictive" mental model of the opponent when we did not ask the subject to predict the opponent's next decision (see Appendix C for our selection criteria of payoff structures). Hedden and Zhang (2002) always asked for a prediction, while we only did so in Test Block 1 for the Prompt group.

structures often fit with having a too simple model of the opponent, using only first-order theory of mind. For example, they expect the opponent to always play safe by staying at B if either of their payoffs in C or D is smaller than the one in B, or to always take a risk by moving on if either of their payoffs in C or D is larger than the one in B. Both these simple models do not take into account that the opponent is also reasoning about the subject's final decision. Based on feedback, many subjects improve their model of the opponent in the course of the experiment and start to correctly make a second-order theory of mind model. Starting out "as simple as possible", they learn to reason "as complex as necessary" (Meijering et al., 2014).

4.2 How do subjects reason when they make optimal decisions?

We surmise that subjects who make optimal decisions do not apply a fixed simple reasoning strategy based on comparison of payoffs only, such as backward induction, without explicitly reasoning about mental states. ¹³ Our attribution of recursive perspective taking to subjects who make optimal decisions is corroborated by a related Marble Drop experiment, in which subjects were also told that they were playing against a self-interested computer opponent (Meijering et al., 2012). Subjects' eye movements show them not to use backward induction. They attend mostly to the trapdoors, the decision points, not only to the payoff pairs, which would have been sufficient for backward induction.

Subjects' eye movements first show predominantly left-to-right progressions and only then, for the more difficult payoff structures, their eyes make right-to-left progressions at the end of reasoning about a game. This fits what we dub the "forward reasoning plus backtracking" reasoning strategy: you start at the first decision point in a game and proceed to the next one for as long as higher outcomes are expected to be available at future decision points; when you discover that you may have unknowingly skipped the highest attainable outcome at a previous decision point, you jump back to inspect whether that outcome is indeed attainable (Meijering et al., 2012).

This temporal order of subjects' reasoning is suggestive of causal reasoning, which they have learnt from early child-hood (Gopnik et al., 2004): First "What would happen if I chose to move on to the right?", then "What would happen if the opponent then chose to move on to the right as well?", and finally "What would I then choose at the final pair of trapdoors? Oh, that would give my opponent the lightest-

colored blue marble, so he probably wouldn't want to let me have the final choice and end up there."

Moreover, when we compare the reaction times with computational models of backward induction and of forward reasoning plus backtracking, it turns out that subjects' patterns of decision times on all game items fit "forward reasoning plus backtracking" much better than backward induction (Bergwerff, Meijering, Szymanik, Verbrugge, & Wierda, 2014). Levidence for such forward reasoning has also been found in some other turn-taking games, such as sequential bargaining and a sequential version of the trust game (Johnson et al., 2002; Evans & Krueger, 2011).

4.3 Comparison with another study supporting second-order perspective-taking

Goodie, Doshi and Young (2012) claim that perspective taking by itself is not the bottleneck in turn-taking games requiring second-order theory of mind. In line with this, they propose two ways to support people in applying second-order ToM in a turn-taking game such as the Matrix game. Their first manipulation is to turn the abstract Matrix Game into a game-theoretically isomorphic version that has a concrete military cover story; this turns out to have no effect. Their second manipulation is to transform the Matrix Game into a competitive fixed-sum game with a single pay-off shown per cell. They argue that in these zero-sum games, adults are perfectly able to apply second-order ToM against a predictive player, with more than 90% correct decisions (Goodie et al., 2012). They argue that such strictly competitive games are more natural for people (as also claimed by (Bornstein, Gneezy, & Nagel, 2002)).

However, at second sight, there is an alternative explanation for their positive findings about the zero-sum game. In their Experiments 1 and 2, all 40 critical trials of their 80-trial test phase displayed the same ordering of payoffs over the four subsequent leaves of the centipede-like decision tree, namely 3–2–1–4; these had been mapped to probabilities with different cardinalities, but in the same order (Goodie et al., 2012). This similarity between items may have enabled their subjects after a number of trials to understand that the solutions all followed the same pattern (similar to our game 3 of Table 2 of Appendix C). The successive three optimal choices are always the same for the zero-sum games: Player 1 stays at A, because Player 2 stays at B, because Player 1 goes on to D.

In our experiment, such pattern recognition could not arise because each subject encountered only 4 spaced instances of

¹³Note that to *apply* backward induction, no theory of mind is needed, but to *discover* it, second-order theory of mind is required (Verbrugge, 2009). We took care that subjects did not have much pre-knowledge of game theory, in particular, none of them had been exposed to backward induction. Interestingly, two subjects told us in the exit interview that, after playing many different game items, they had discovered a "neat trick" that saved them time. That trick turned out to be backward induction.

¹⁴Decision times were strongly correlated to the complexity of payoff structures: for game items in which it was sufficient to reason through the game from the beginning to the end by forward reasoning without having to do backtracking, such as game 16 of Table 9 (Appendix C), subjects' reaction times were significantly shorter than for items in which backtracking was needed, such as game 2.

each of the 16 payoff structures, and all these 16 of our payoff structures were essentially different (see Table 2 of Appendix C). Also, in real life, many important social situations requiring second-order ToM cannot be re-cast as zero-sum games, but only as general-sum games, so the Matrix Game or Marble Drop appear to be better ways to train people in second-order perspective taking for real-life intelligent interactions.

4.4 Implications: De-biasing decision makers

The findings of the current study, in particular the notion of scaffolding decision making for subjects by stepwise training, could also be of interest in attempting to reduce the biases of decision makers in other social situations. For example, people often allocate resources in a parochial way. Without explicitly wanting to harm out-groups, people reduce their own earnings in order to support their own ingroup (e.g., their nation), even if their decision is so detrimental to an out-group (e.g., another nation) that the overall outcome of their decision is negative, while it could have been positive (Bornstein & Ben-Yossef, 1994; Baron, Ritov, & Greene, 2013). One way to reduce this type of in-group bias is to change the task representation by asking subjects to compute all gains and losses (Baron, 2001, Experiment 2). In view of the results of the current study, it might alternatively be helpful to offer the subjects a stepwise training in which they have to make allocation decisions of increasing complexity and with growing numbers of parties. After each non-optimal allocation decision in the stepwise training block, subjects could be shown which other decision would have had a more positive overall outcome and they could be asked to explain why that alternative decision is better. It would be interesting to compare the effects of these two de-biasing methods, asking for explicit computations and stepwise training, in a future experiment.

A related phenomenon is the bias towards self payoffs in sequential trust games, where people appear not to be sufficiently interested in perspective-taking to even gather information about the opponents' payoffs, often to their own detriment (Evans & Krueger, 2014). Another type of bias is the false consensus phenomenon, in which people think that their own judgments and decisions are more prevalent in their community than is actually the case (Ross, Greene, & House, 1977). Related to this, the "Isn't everyone like me?" bias occurs in a normal form game in which subjects who choose one action ego-centrically tend to believe that other players will choose that same action in a larger proportion than is expected by subjects who choose the other action (Rubinstein & Salant, 2016). It would be interesting to investigate whether these biases can also be alleviated by step-by-step training in properly taking other people's perspectives into account.

4.5 Conclusion

In two-player turn-taking games with three decision points, reasoning about someone else's decision that in turn depends on your own plan is difficult, even for adults. The required second-order perspective taking ("the opponent thinks that I would intend to go left at the last decision point") does not appear to happen automatically or spontaneously. However, our results have shown that subjects in such turn-taking games can learn to take the required perspective to make optimal decisions when they are knowingly playing against a self-interested opponent taking the subject's goal into account.

From a theoretical perspective, it would seem that subjects could be supported by three manipulations, separately or in combination: 1) step-wise training, in which they are subsequently trained to make and explain decisions in games with one, two and three decision point that require zero-order, first-order and second-order theory of mind, respectively; 2) prompting subjects to predict what the opponent would choose at the next decision point before making their own decision at the first decision point; and 3) using an intuitive visual interpretation, namely Marble Drop, which is closer to people's daily experiences than the Matrix Game.

It turns out that, while prompting for predictions has largely a short-time effect and the visual task presentation does not make much of a difference, there is a clear positive main effect of stepwise training. Thus, stepwise training can be used to teach people to recursively put themselves in other people's shoes, which is important in many social interactions, from the simple turn-taking Matrix Game to international peace negotiations.

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Appendix A: Instructions

Here follows a sample set of instructions as presented verbatim to subjects, all of them given in the form for the Marble Drop task representation. First we present the instructions for Stepwise training, then the instructions for the test blocks for the Prompting group. The instructions for other conditions are similar.

Instructions for Step-wise training

The subjects first received the instructions for zero-order games on the screen, after which they practiced four zero-order games. Then they were shown the instructions for first-order games, after which they practiced eight different first-order games. Finally, they received the instructions for the second-order games on the screen, after which they went through eight second-order practice games.

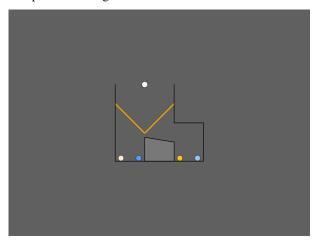
Instructions zero-order games

In this task, you will be playing marble games such as the one in the figure below.

A white marble is about to drop and you can change its path by removing one of two orange trapdoors. The white marble has to end up in the bin with the darkest orange marble.

Press \backslash to remove the left trapdoor, and / to remove the right trapdoor.

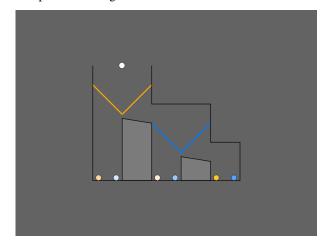
Press spacebar to begin.



Instructions first-order games

In the next games, there are two sets of trapdoors. You control the orange trapdoors and the computer controls the blue trapdoors. The computer is trying to let the white marble end up in the bin with the darkest blue marble it can get. You still have to attain the darkest orange marble you can get. Note, the computer does not play with or against you.

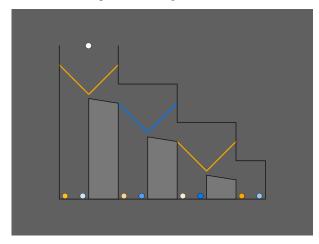
Press spacebar to begin.



Instructions second-order games

In the next games, there are three sets of trapdoors. Again, you control the orange trapdoors, and the computer controls the blue trapdoors. Note, for its decision, the computer takes into account what your choice will be at the third set of trapdoors. Remember, the computer does not play with or against you.

Good luck! Press spacebar to begin.



Instructions at the start of the Test Blocks

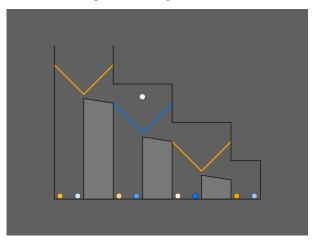
After the training items, the subjects were shown new instructions at the start of the first and the second Test Block. Below, we present the instructions at the beginning of Test Block 1 and Test Block 2 for the Prompt group.

Instructions second-order prediction-decision games: Test Block 1

The next games are slightly different. First, you have to predict what the computer will do at the second set of trapdoors. Secondly, you have to decide what to do at the first set of trapdoors.

To speed up the experiment, the animations are removed from the games.

Good luck! Press spacebar to begin.



Instructions second-order prediction-decision games: Test Block 2

In the next games, you do not have to predict anymore what the computer will do at the second set of trapdoors. You immediately receive feedback after you have decided what to do at the first set of trapdoors.

Good luck! Press spacebar to begin.

Appendix B: the $2 \times 2 \times 2$ groups and allocation of subjects

Altogether, there were 93 subjects. Per manipulation, here are the totals of subjects:

- Training: Stepwise training 46, Undifferentiated training 47;
- Prompting for predictions: Prompt 47, No-Prompt 46;
- Task representation: 43 Marble Drop, 50 Matrix Game.

Table 1 gives the numbers of subjects per group.

Appendix C: Pay-off structures

Tables 2, 3, 4, and 5 represent the relevant payoff structures, together with corresponding correct decisions and predictions for our experiment. Letters A, B, C and D stand for the

Table 1: The 2 x 2 x 2 experimental groups and numbers of subjects for each

group nr.	Training	Prompting	Task Representation	number of subjects
1	Stepwise	Prompt	Marble Drop	10
2	Stepwise	Prompt	Matrix Game	13
3	Stepwise	No- Prompt	Marble Drop	10
4	Stepwise	No- Prompt	Matrix Game	13
5	Undifferen- tiated	Prompt	Marble Drop	12
6	Undifferen-	Prompt	Matrix Game	12
7	Undifferen- tiated	No- Prompt	Marble Drop	11
8	Undifferentiated	No- Prompt	Matrix Game	12

bins in the Marble Drop game from left to right, respectively for subsequent cells in the Matrix game. Each payoff pair represents the payoffs of Player 1 and Player 2, respectively, where 1 represents the lowest payoff (corresponding to the lightest shade of their target color in Marble Drop), while 4 represents the highest payoff (corresponding to the darkest shade of their target color in Marble Drop).

Second-order payoff structures for games with three decision points

The following computation has been adapted from Section 2.1.2 of (Hedden & Zhang, 2002). In principle, there are $24 \times 24 = 576$ different payoff structures for three-decision Matrix and Marble Drop games, on the basis of dividing pairs of payoffs from 1, 2, 3, 4 for both players over four cells. However, by far not all of these 576 games would be suitable for testing subjects' second-order ToM in a turntaking game. Let us describe our exclusion criteria by which we selected the 16 different games that we used for game items in the two test blocks as well as in the final 8 games of Stepwise Training.

Excluding payoff structures requiring only zero-order

ToM. Payoff structures are excluded if Player 1's payoff in A is either a 1 or a 4, because Player 1 would not need to reason about Player 2's decision at all. It is obvious that Player 1 should continue the game if his payoff in A is a 1 and stop if his payoff in A is a 4. The game in Figure 8 (right) is an example in which Player 1 should immediately decide to

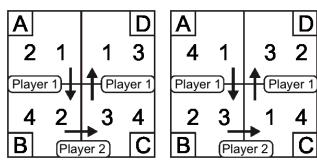


FIGURE 8: Two trivial Matrix Games: The game on the left is a so-called trivial first-order game. See text for explanation. The game on the right does not require any ToM reasoning at all, because Player 1's maximum payoff is already available in cell A.

stop in A. Therefore, in line with Hedden and Zhang (2002), we focused on so-called 2- and 3-starting games, associated with payoff structures in which Player 1's first payoff was a 2 or a 3, respectively.

Excluding "trivial" payoff structures requiring only first-order ToM. Of the remaining payoff structures, we excluded the so-called trivial ones in which Player 2's payoff in B was either lower or higher than both his payoffs in C and D. The left matrix of Figure 8 depicts an example of such a game: Player 2 does not need to reason about Player 1's last possible decision, as his payoffs in C and D are both more preferable than his payoff in B. This included payoff structures in which Player 2's payoff in B was either a 1 or a 4, because Player 2 would not need to reason about Player 1's last possible decision between C and D. Accordingly, first-order reasoning on the part of Player 1 would suffice.

Excluding payoff structures that do not distinguish between attributed opponent types. The next two exclusion criteria are based on the type of Player 2 that a subject could be reasoning about. A subject, always assigned to the role of Player 1, might be reasoning about a zero-order Player 2 who would not reason about the subject's last possible decision. Hedden and Zhang call such a Player 2 "myopic" (short-sighted), because they only consider their own payoffs in B and C. In contrast, a subject might be reasoning about an hypothesized first-order or "predictive" Player who does reason about ("predict") the subject's last possible decision.

Because Player 1's decision at A depends on Player 2's decision at B, payoff structures that yield the same answer for a zero-order and a first-order Player 2 cannot inform us about the level of ToM reasoning on the part of Player 1. We consider these payoff structures to be non-diagnostic, and therefore we exclude them from the final set of stimuli. Hedden and Zhang, who prompted each subjects for predictions of Player 2's choice at B, could include such payoff structures as long as the correct prediction of Player 2's move at the

Table 2: Payoff structures of second-order games, adapted from (Hedden & Zhang, 2002)

ID	Payoff pairs Players 1, 2				Correct prediction Pl. 2 at B	Optimal decision Pl. 1 at A
	A	В	С	D		
1	3, 2	1,3	2, 4	4, 1	stay	stay
2	3, 4	1, 2	2, 3	4, 1	stay	stay
3	3, 1	2, 3	1, 4	4, 2	stay	stay
4	3, 4	2, 2	1, 3	4, 1	stay	stay
5	3, 1	4, 3	1, 2	2, 4	go	stay
6	3, 2	4, 3	1, 1	2, 4	go	stay
7	3, 3	4, 2	1, 1	2, 4	go	stay
8	3, 4	4, 2	1, 1	2, 3	go	stay
9	3, 1	4, 3	1, 4	2, 2	stay	go
10	3, 2	4, 3	1, 4	2, 1	stay	go
11	3, 3	4, 2	1, 4	2, 1	stay	go
12	3, 4	4, 2	1, 3	2, 1	stay	go
13	3, 2	1, 3	2, 1	4, 4	go	go
14	3, 4	1, 2	2, 1	4, 3	go	go
15	3, 1	2, 3	1, 2	4, 4	go	go
16	3, 4	2, 2	1, 1	4, 3	go	go

second decision point was opposite for imagined zero- and first-order Player 2s. In contrast, we prompted only half of our subjects (the Prompt group) for predictions in the first test block and none of them in the second test block, so we had to exclude all these items.

Balancing the decisions and predictions between "stop" and "go". We selected a final set of stimuli, which we were able to (double-)balance for both the number of optimal *stay* and *go* decisions of Player 1 and the number of optimal *stay* and *go* decisions of Player 2. Because this was only possible for 3-starting games, we excluded the 2-starting games. The final balancing left us with 16 unique payoff structures (Table 2).

Stepwise training

Let us now present the payoff structures for the training games in the *Stepwise training* condition, in which the subjects are first presented with 4 zero-order games, then 8 first-order games, and finally 8 second-order games. As a reminder, zero-order games have one decision point with only the two possibilities A and B; first-order games have two decision points and the subsequent possibilities A, B, and C; finally, second-order games have three decision points and the subsequent possibilities A, B, C, and D.

Table 3: Payoff structures of zero-order games with one decision point. Each payoff pair represents the payoffs of Player 1 and Player 2, respectively

Payoffs P	layer 1 and 2	Optimal decision Player 1 at A	
A	В		
1, 3	3, 2	go	
3, 2	1, 1	stay	
4, 2	2, 4	stay	
3, 2	4, 4	go	

Zero-order training games

The four payoff structures used in our zero-order training games are presented as four rows in Table 3. The final column represents for each of the four payoff structures the optimal decision for Player 1 at the decision point: *stay* at A or *go* on to B. Note that we have chosen the sets of payoff structures such that optimal ("correct") decisions are balanced between *stay* and *go* over the zero-order games.

First-order training games

The fourth column in Table 4 represents for each of the 8 first-order payoff structures (rows in the table) the corresponding correct prediction about Player 2's decision at the second decision point: *stay* at B or *go* on to C. The fifth column represents the optimal "correct" decisions of Player 1 at the first decision point: *stay* at A or *go* on to B.

We have chosen the 8 first-order payoff structures such that they are all diagnostic of first-order ToM for Player 1, in the sense that Player 1's optimal decision to *stay* or *go* at the first decision point depends on correctly predicting the decision of a rational Player 2 at the second decision point. That is to say, Player 1's payoff in A is not larger than both of Player 1's payoffs in B and C, nor is it smaller than both Player 1's payoffs in B and C; instead, it is in between those payoffs, therefore, it really matters whether Player 2 will choose to stay at B or go on to C.

Note that we have chosen the sets of payoff structures such that both correct predictions and correct decisions are balanced between *stay* and *go* over all first-order game items.

Second-order training games Table 2 presents the 16 diagnostic second-order games of which 8 are selected in the last step of step-wise training.

Undifferentiated training

Table 5 presents 24 trivial payoff structures. These games have the same three decision points as the second-order games. However, they only require first-order ToM for Player 1 to make the correct decision, because Player 2 can make

Table 4: Payoff structures of first-order games with two decision points. Each payoff pair represents the payoffs of Player 1 and Player 2, respectively

Payoff pairs Players 1, 2			Correct prediction Player 2 at B	Optimal decision Player 1 at A
A	В	С		
2, 1	1, 2	3, 3	go	go
2, 3	1, 2	3, 1	stay	stay
2, 1	3, 2	1, 3	go	stay
2, 3	3, 2	1, 1	stay	go
3, 2	2, 3	4, 4	stay	go
3, 4	2, 3	4, 2	stay	stay
3, 2	4, 3	2, 4	go	stay
3, 4	4, 3	2, 2	go	stay

an optimal decision without taking Player 1's final decision into account.

For example, if the game of row (a) gets to B, Player 2 will always move from B to C regardless of what Player 1 would choose at the third decision point, because for Player 2, both payoff 4 in C and payoff 3 in D are better than his payoff in B. Therefore, Player 1 should predict that Player 2 will *go* on to C. On the basis of this prediction, Player 1 can safely *go* on to B, and at the final decision point, Player 1 can *stay* at C to get the optimal attainable payoff, namely 3.

Table 5: Payoff structures of trivial first-order games with three decision points. This table has been adapted from Appendix B of (Hedden & Zhang, 2002)

ID	Payor	ff pairs	Player	s 1, 2	Correct prediction Pl. 2 at B	Optimal decision Pl. 1 at A
	A	В	С	D		
a	2, 1	4, 2	3, 4	1, 3	go	go
b	3, 1	4, 2	2, 3	1, 4	go	stay
c	2, 4	1, 3	4, 1	3, 2	stay	stay
d	3, 4	4, 3	1, 1	2, 2	stay	go
e	2, 1	3, 2	4, 3	1, 4	go	go
f	3, 4	4, 3	1, 2	2, 1	stay	go
g	2, 4	1, 3	3, 1	4, 2	stay	stay
h	3, 1	4, 2	2, 4	1, 3	go	stay
i	3, 4	4, 3	2, 2	1, 1	stay	go
j	2, 4	1, 3	4, 2	3, 1	stay	stay
k	3, 1	4, 2	1, 4	2, 3	go	stay
1	2, 1	4, 2	3, 3	1, 4	go	go
m	3, 4	4, 3	2, 1	1, 2	stay	go
n	3, 1	4, 2	1, 3	2, 4	go	stay
o	2, 1	3, 2	4, 4	1, 3	go	go
p	2, 4	1, 3	3, 2	4, 1	stay	stay
q	3, 4	2, 3	4, 1	1, 2	stay	stay
r	2, 4	4, 3	1, 2	3, 1	stay	go
S	2, 1	1, 2	4, 3	3, 4	go	go
t	3, 4	4, 1	1, 2	2, 3	go	stay
u	2, 1	1, 2	3, 4	4, 3	go	go
v	2, 4	3, 3	1, 2	4, 1	stay	go
w	3, 4	4, 1	2, 2	1, 3	go	stay
X	3, 4	1, 3	4, 2	2, 1	stay	stay