Valuing bets and hedges

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Abstract

Two studies replicate the anomaly identified by Frederick, Meyer and Levis (2015) and Frederick, Levis, Malliaris and Meyer (2018). People show typical risk averse behavior by valuing risk below the focal lottery’s expected value, but they do not bid above its expected value for the hedge that eliminates the risk. Following the authors, we conduct finer analyses by separating participants into two groups – “experts” who understand that acquiring the hedge makes winning certain versus “novices” who do not understand the winning implications of acquiring the hedge. We find that (1) “experts” are more inclined to purchase the hedge compared to the “novices” and (2) unlike the “novices,” they value the hedge significantly more than the risk instrument, but only if they are given the risk instrument free of charge. However, even there, the hedge valuations are significantly less than the lottery’s expected value suggesting that the anomaly described in Frederick et al. (2015, 2018) is robust and likely to affect the way our discipline conceptualizes and models risk behavior.

Keywords: bets, hedge, risk, willingness to pay

1 Introduction

Risk is a part of people’s daily lives. When people take risks, they reap rewards sometimes and suffer losses at other times, and, fearing the latter, they often hedge against the risk. For example, a person who has invested in automobile stocks may fear that her investment will lose value if the economy goes down, and consequently, she may buy defensive stocks (e.g., food, utilities that typically go up or maintain value during downturns) as a hedge against her automobile investments. However, whereas there is extensive research on how people value risk, and if they behave consistently when they do so (e.g., Prospect Theory, Kahneman and Tversky, 1979), there is very little research that compares people’s value of risks to their value of hedges. Two exceptions are Frederick, Meyer and Levis (2015) and Frederick, Levis, Malliaris and Meyer (2018), who show that people are, normatively speaking, inconsistent in how they value the two. To illustrate, consider the following example from Frederick et al. (2015):

Suppose that a person wishes to enter a lottery that wins her $10 on a coin flip (Heads or Tails). To enter the lottery, she can buy a Heads voucher (guaranteeing her the win if the coin lands Heads) or a Tails voucher (guaranteeing her the win if the coin lands Tails). Imagine that she wishes to bet Heads and therefore buys a Heads voucher (the risk instrument) for (say) $3. The $3 captures how she values the risk, i.e., the net result of her trade-off between the desirable outcome (winning $10) and the undesirable outcome (winning nothing). However, now that she has acquired a Heads voucher, suppose that we ask her how much she is willing to pay for a Tails voucher. Notice that her acquiring a Tails voucher reduces her risk to zero, i.e., she is fully protected or hedged against the risk. Frederick et al. (2015, 2018) specify that the normatively correct or consistent way of valuing the Tails voucher (hedge instrument) is that the sum of the two valuations (risk and hedge instruments) should equal to the prize amount ($10). In other words, people who bid below the lottery’s expected value for the risk instrument should bid above its expected value for the hedge. However, their studies show that people’s valuation of the two instruments seldom add up to the prize amount.

We report two studies replicating the Frederick et al. (2015, 2018) results. Following the authors, we do finer analyses by separating participants into two groups – “experts”, who understand that acquiring the hedge makes winning certain versus “novices,” who do not understand the winning implications of acquiring the hedge. We find that (1) experts are more inclined to purchase the hedge compared to novices and (2) unlike the novices, they value the hedge significantly more than the risk instrument but only when they are given the risk instrument free of charge. However, even then, they do not value the hedge above the lottery’s expected value.

1.1 Normative Answer

Frederick et al. (2015) describe the normative answer to the risk plus hedge valuation in the following way. When a per-
son bids a certain amount, say $3, for the risk instrument (say the Heads voucher), she is deducting $7 from the winning amount ($10–$3) to compensate for the undesirable part of the lottery (getting nothing in return). However, when she values the hedge that takes away the undesirable part of the lottery completely (i.e., there is no chance now that she will lose), normatively speaking, she should be willing to give back what she had deducted in the first place (to compensate her for taking the risk). This would imply that she bid above the expected value of the lottery to acquire the Tails voucher ($7). This logic, as put forward by Frederick et al. (2015, 2018), implies that the value of the Heads and Tails voucher should equal to $10 and the risk and hedge values should be perfectly and negatively correlated at –1.

1.2 Testing for the Risk/Hedge Distinction

In our studies, we follow the lottery example of Frederick et al. (2015) and consider two issues. We note that the original studies of Frederick et al. (2015, 2018) considered these issues as well, and this is a replication of their work.

First, we consider whether people have the correct understanding of the probabilities of winning associated with the risk and hedge instruments. If we expect people’s valuation of the risk and hedge instruments to be normatively consistent, we have to assume that they correctly understand that the probability of winning the lottery once they have acquired the hedge instrument is 100%. However, as research suggests, people are, generally speaking, not very good at understanding probabilities. This is true for single event probabilities (e.g., what is the probability of winning the lottery if you buy the Heads voucher or the Tails voucher) or conditional probabilities that (what is the probability of winning the lottery if you buy a Tails voucher, given that you already possess a Heads voucher; see, for example, Gigerenzer and Edwards, 2003).

There are various reasons why people may have an imperfect understanding of probabilities – for example, people may have “intuitive” methods of thinking that interfere with the learning of correct statistical reasoning (Denes-Raj & Epstein, 1994; Garfield & Ahlgren, 1988; Szaszi et al., 2018). In our case, it is not so much important to understand why people cannot figure that acquiring the hedge reduces risk to zero. However, it is important to understand (and control for) the fact that, if people do not understand that the hedge instrument makes winning certain, then they are likely to value the hedge as just another risk instrument. Accordingly, in both our studies, we quiz the participants on their understanding of the probability of winning the lottery when they have acquired both the Heads and Tails vouchers.

Second, we consider the implications of consumer surplus on people’s valuations. When people indicate a willingness-to-pay (WTP) number, they normally wish to keep a surplus for themselves (for example, to get transaction utility or a good deal; Thaler, 1985). In our case, when a person enters a lottery she desires to win some money, and she may balk at paying the full prize amount to be completely hedged against the risk. This problem arises in a within-subjects environment where people indicate a WTP for the risk instrument and then a WTP for the hedge instrument (i.e., they pay twice for the same lottery).

We try two things to obviate this problem. First, in Study 1, we measure attractiveness ratings of the risk and hedge instruments (in addition to WTP) since attractiveness ratings do not carry any surplus implications. Second, in Study 2, we ask participants to imagine that they have acquired the risk instrument free of charge before we ask them to value the hedge instrument. In this way, we try to minimize any feelings that they are paying twice for the same prize.

2 Study 1

2.1 Method

In Study 1, 90 MTurk volunteers imagined entering a (fair) coin toss lottery with a $10 prize. We told them that they could buy a Heads voucher or a Tails voucher, explaining to them that buying the Heads (Tails) voucher would guarantee them the win if the coin landed Heads (Tails). We asked participants (1) their WTP for a Heads voucher and (2) to rate the attractiveness of the Heads voucher (9-point unattractive/attractive scale). Then we asked them to indicate their attractiveness rating/WTP for a Tails voucher, reminding them explicitly that they already possessed a Heads voucher. Finally, we asked them how likely they were to win the lottery now that they had a Heads voucher and a Tails voucher.

2.2 Analysis and Results

Sixty (out of 90) participants could correctly indicate that the probability of winning the lottery was 100% with the Heads and Tails vouchers. Accordingly, we divided the sample into two groups, (1) experts, or those who correctly understood their probability of winning the lottery was 100% once they acquired the hedge (n = 60) and (2) novices, or those who thought that it was less than 100% (n = 30).

The attractiveness ratings for the risk and hedge instruments were positively correlated at the aggregate (r = 0.23, p = 0.03), more strongly so among the novices (r = 0.46, p = 0.01) than the experts (r = 0.21, p = 0.10; the difference between the two correlations was not significant). Relative to the novices, the experts found the risk instrument less attractive, and the hedge instrument more attractive, but neither of the two differences was statistically significant (risk: M’s of 5.20 and 5.50; F(1, 88) = 1.01, p = 0.32; hedge: M’s of 5.92 and 5.37; F(1, 88) = 3.26, p = 0.07).
We conducted a repeated-measures ANOVA with attractiveness of the Heads and Tails vouchers as the repeated measures factor and group (experts, novices) as the between-subjects factor and found a significant interaction between the two (F(1, 88) = 5.35, p = 0.02). While experts found the Tails voucher (hedge) to be significantly more attractive than the Heads voucher (risk; M’s of 5.92 and 5.20; F(1, 59) = 8.82, p = 0.005), novices found the two to be equally attractive (M’s of 5.37 and 5.50; F(1, 29) < 1).

The WTP measures for the risk and hedge instruments were positively correlated at the aggregate (r = 0.50, p < .0001), more strongly so among the novices (r = 0.84, p < 0.0001) than the experts (r = 0.38, p = 0.01; p < .001 for the difference). The experts were willing to pay more for the risk instrument and the hedge instrument compared to the novices, but neither of the two differences was statistically significant (risk: M’s of 2.50 and 2.00; F(1, 88) = 1.90, p = 0.17; hedge: M’s of 2.60 and 2.03; F(1, 88) = 1.95, p = 0.17). Summed across the two vouchers, the experts were willing to pay more than the novices were to acquire both the Heads and Tails vouchers, but, once again, the difference was not statistically significant (M’s of 5.10 and 4.03; F(1, 88) = 2.59, p = 0.11). Moreover, we should note that neither of the two sums is anywhere close to the total prize amount ($10).

As with the attractiveness measures, we conducted repeated-measures ANOVA with the WTP measures, but we did not find any significant effects. The interaction between the within subjects factor (WTP for the Heads and Tails vouchers) and the between subjects factor (experts, novices) was not significant (F(1, 88) < 1). Among the experts and novices, the WTP for the Heads voucher (risk) and the Tails voucher (hedge) were the same (Experts: M’s of $2.50 and $2.60; F(1, 59) < 1; Novices: M’s of $2.00 and $2.03; F(1, 29) < 1).

### 2.3 Discussion

The results of Study 1 suggest that experts find the hedge instrument significantly more attractive than the risk instrument, but they are not willing to pay significantly more to acquire it. We speculate that this may be due to surplus considerations, i.e., the participants do not wish to pay twice for the lottery (sacrificing their surplus along the way), an issue that we consider in Study 2.

### 3 Study 2

#### 3.1 Method

One hundred two M-Turk volunteers participated in Study 2. We used the same stimuli materials as in Study 1. The volunteers imagined entering a (fair) coin toss lottery with a $10 prize and we explained how they could win the prize with a Heads or Tails voucher (see Study 1). At this point, we asked them if they preferred to buy a Heads voucher or a Tails voucher and indicate how much they were willing to pay for a voucher of their choice. Thus, and unlike Study 1, we did not force them to value the Heads voucher first, and whichever voucher they chose (Heads to Tails) became (for the purpose of our analysis) the risk instrument. Thereafter, we told them that a friend had given them their voucher of choice, free of charge, and now they could buy the other voucher if they wished to do so. At this point, we asked them how likely they were to buy the other voucher (9-point, “very likely” to “not at all likely” scale), and how much they were willing to pay for that voucher. For the purpose of our analysis, this second voucher is our hedge instrument. Finally, we asked them to indicate how likely they were to win the lottery now that they possessed a Heads voucher and a Tails voucher.

#### 3.2 Analysis and Results

Fifty-five (out of 102) participants, or 54%, correctly indicated that the probability of winning the lottery was 100% when they had the Heads voucher and the Tails voucher. As in Study 1, we classified these participants as experts and we classified the rest as novices. Among experts (novices), 76% (89%) chose the Heads voucher to enter the lottery. However, including their choice (Heads or Tails) in our analyses (reported below) does not change the results, and we do not discuss this issue further.

The WTP measures for the risk and hedge instruments were positively correlated at the aggregate (r = 0.71, p < .0001), more strongly so among the novices (r = 0.91, p < 0.0001) than the experts (r = 0.55, p < 0.0001; p < .0001 for the difference). The experts were willing to pay less for the risk instrument and more for the hedge instrument compared to the novices, but neither of the two differences was statistically significant (risk instrument: M’s of 2.48 and 3.26; F(1, 100) = 2.16, p = 0.14; hedge instrument: M’s of 3.63 and 3.00; F(1, 100) = 1.19, p = 0.28). Summed across the two vouchers, the experts were willing to pay less to acquire the Heads and Tails vouchers, but, once again, the difference was not statistically significant (M’s of 6.11 and 6.26; F(1, 100) < 1). As in Study 1, neither of the two sums is anywhere close to the total prize amount ($10).

The experts and the novice differed significantly on one measure, which is, their likelihood to buy the hedge instrument knowing they have the risk instrument free of charge. Experts were significantly more likely to buy the hedge compared to novices (M’s of 7.78 and 5.62; F(1, 100) = 4.72, p < 0.0001), and it is likely that this difference arose because experts correctly realize that have a sure chance of winning the lottery with the hedge.

We ran repeated-measures ANOVA with the WTP for the risk and hedge instruments as the repeated measures factor
and group (experts, novices) as the between-subjects factor, treating the likelihood of buying the hedge as a covariate. Unlike Study 1, we found a significant interaction between the repeated measures and the between subjects factor (F(1, 99) = 4.37, p = 0.04). Experts were willing to pay significantly more for the hedge instrument than the risk instrument (M’s of $3.63 and $2.48; F(1, 54) = 11.96, p = 0.001) whereas the novices valued the two about the same (M’s of $3.00 and $3.26; F (1, 46) = 1.82, p = 0.18). However, we should note that experts value both the risk and the hedge significantly less than the lottery’s expected value (risk: t(54) = 8.11, p < 0.0001; hedge: t(54) = 3.66, p = 0.001).

For our final analysis, we estimate to what extent the (differential) likelihood to acquire the hedge mediates the hedge/risk valuations of the two groups (value of the hedge instrument minus the value of the risk instrument) using Hayes’ process model (Model 4; Hayes, 2018). The process tests show that experts are more likely to buy the hedge compared to novices (b = 2.16, t = 4.97, p < 0.0001), and the latter, in turn, is significantly associated with the hedge/risk valuations (b = 0.23, t = 2.56, p = 0.01). Tests for the direct/indirect paths shows (1) a significant direct path linking experts/novices to the hedge/risk valuation (direct effect = 0.91, t = 2.09, p = 0.04) as well as (2) a significant indirect path where the likelihood of buying the hedge mediates the direct effect (indirect effect = 0.49, 95% bootstrapped confidence interval = 0.19, 0.88).

### 3.3 Discussion

In Study 2, we find that experts, unlike novices, value the hedge instrument significantly more than the risk instrument, a difference that we did not observe in Study 1. The difference between the two studies is that in Study 2 the participants had the risk instrument free of charge. We should note that we observed the same pattern in the attractiveness ratings of the risk and hedge instruments in Study 1 (ratings that are unaffected by surplus implications). However, we need more research before we can conclude that the significant difference arises only out of surplus considerations.

### 4 General Discussion

Our research replicates the results of Frederick et al. (2015, 2018) and draws attention to two areas. First, researchers may wish to look more carefully into the valuations risks and hedges separately for those who correctly understand that acquiring the hedge makes winning certain (experts) and those who do not (novices). For example, if the fundamental premise is that certainty should be valued more than risk, then we have to ascertain, first, that the participants understand that acquiring the hedge makes winning certain.

Second, even when acquiring the hedge makes winning certain, we need to consider the averseness of having to pay twice for the lottery and the impact that it may have when people value the hedge after they have valued the risk. Here, we propose that, in order to have a true comparison between a risk valuation and a hedge valuation, we need to figure out a way to ensure that each is a single purchase related to the lottery. In Study 2, for example, we have people value the risk instrument first, and then ask them to imagine that they get the risk instrument free of charge before we ask them to value the hedge instrument.

We conclude by addressing two issues. First, what are the practical implications of our research? While we, like Frederick et al. (2015, 2018), find that people deviate from normative expectations in their valuations of risk and hedge instruments, there is a bigger issue here. Hedges, after all, protect people from the downside of taking risks and an improper valuation of the hedge may deny people from acquiring this protection. For example, given that risk averse consumers should get the most benefit from hedging their bets (Webster, Paltsev & Reilly, 2010), it is important that they do not set too low a value to the hedge, such that the market denies them the risk-reducing instrument. Thus, future research may look at how consumers with different risk profiles value hedges and what we can do to sensitize consumers to their value in eliminating risk.

Second, in our research we test for simple lotteries. This implies that acquiring a hedge instrument reduces the risk to zero (it makes winning certain). However, real world bets are seldom so simple. For example, one can think of a product introduction scenario where we have, say, a 30% chance of making large profits, a 20% chance of making small profits, another 20% chance of breaking even, and a 30% chance of making a loss. Here, if we hedge against incurring a loss, acquiring the hedge will still not guarantee that we will make large profits. How hedge values change depending upon the number of possible outcomes is an interesting avenue for future research.

### References


Frederick, S., Levis, A., Malliaris, S. & Meyer, A. (2018) Valuation of bets and hedges: Implications for the con-


